Consider for $x+y+z=1$, where $x,y,z\in\mathbf{R}^{+}$,

$$\frac{3}{16}\left[(1-x)(1-y)(1-z)\right]^{\frac{4}{3}}$$
$$=\frac{3}{16}\left[1-(x+y+z)+xy+yz+zx-xyz\right]^{\frac{4}{3}}$$
$$=\frac{3}{16}\left[xy+yz+zx-xyz\right]^{\frac{4}{3}}$$
$$\geq\frac{3}{16}\left[3(xyz)^{\frac{2}{3}}-xyz\right]^{\frac{4}{3}}$$
$$\geq\frac{3}{16}\left[3(xyz)^{\frac{2}{3}}-\frac{1}{3}(xyz)^{\frac{2}{3}}\right]^{\frac{4}{3}}$$
$$=\frac{3}{16}\left[\frac{8}{3}(xyz)^{\frac{2}{3}}\right]^{\frac{4}{3}}$$
$$=\frac{1}{3^{\frac{1}{3}}}(xyz)^{\frac{8}{9}}$$
$$\geq(xyz)^{\frac{8}{9}+\frac{1}{9}}$$
$$=xyz$$

Put $x=\dfrac{a}{a+b+c}$, $y=\dfrac{b}{a+b+c}$, $z=\dfrac{c}{a+b+c}$

$$\frac{3}{16}\left[\left(\frac{b+c}{a+b+c}\right)\left(\frac{c+a}{a+b+c}\right)\left(\frac{a+b}{a+b+c}\right)\right]^{\frac{4}{3}}\geq\frac{a}{a+b+c}\times\frac{b}{a+b+c}\times\frac{c}{a+b+c}$$
$$\frac{3}{16}\left[(a+b)(b+c)(c+a)\right]^{\frac{4}{3}}\geq abc(a+b+c)$$

$$\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]\geq 0$$

$$\frac{1}{2}\left[a^2-2ab+b^2+b^2-2bc+c^2+c^2-2ca+a^2\right]\geq 0$$

$$a^2+b^2+c^2\geq ab+bc+ca$$

$$(a+b+c)^2\geq 3(ab+bc+ca)$$

$$(ab+bc+ca)^2(a+b+c)^2\geq 3(ab+bc+ca)^3$$

$$\sqrt[3]{\frac{(ab+bc+ca)(a+b+c)}{9}}\geq\sqrt{\frac{ab+bc+ca}{3}}$$

$$\sqrt[3]{\frac{(ab+bc+ca)(a+b+c)-\frac{1}{9}(ab+bc+ca)(a+b+c)}{8}}\geq\sqrt{\frac{ab+bc+ca}{3}}$$

$$\sqrt[3]{\frac{(ab+bc+ca)(a+b+c)-abc}{8}}\geq\sqrt{\frac{ab+bc+ca}{3}}$$

$$\sqrt[3]{\frac{(a+b)(b+c)(c+a)}{8}}\geq\sqrt{\frac{ab+bc+ca}{3}}$$